Immune Based Hybrid Evolutionary Algorithm for Pareto Engineering Optimization

C. J. Shih* and T. L. Kuan

Department of Mechanical and Electro-Mechanical Engineering, Tamkang University, Tamsui, Taiwan 251, R.O.C.

Abstract

This paper proposes an evolutionary multi-objective optimization algorithm that applies the concept of biological immune system as an alternative algorithm for solving Pareto engineering optimization problems. The optimization algorithm developed and presented in this paper uses the cycle of affinity-maturation principle in the immune system that contains the repeated activation, proliferation and differentiation. The algorithm uses the enhanced expression strategy for handling constraints and the recombination in genetic algorithm to promote the solution performance. The designs of Pareto front can be generated in a single run of simulation by applying normalized function and weighting technique. All computational works completed in this paper uses the real-number-coded representation for genes evolution that can be efficiently applied to general engineering design optimization problems.

Key Words: Evolutionary Algorithm, Immune System, Engineering Optimization, Multi-Objective Optimization, Pareto Front Design

1. Introduction

A multi-objective optimization problem contains several parallel objective functions, written as $\{f_1(\boldsymbol{X}), f_2(\boldsymbol{X}), \ldots, f_M(\boldsymbol{X})\}$, are simultaneously minimized subject to constraints of Eqs. (1) and (2):

$$g_i(\boldsymbol{X}) \le 0,\ i = 1, 2, \ldots, m \tag{1}$$

$$\boldsymbol{X}^L \le \boldsymbol{X} \le \boldsymbol{X}^U \tag{2}$$

where $g_i(\boldsymbol{X})$ represents the ith inequality function that represents the general constrained form in design optimization problems. All variables must be evolved for decision making are restricted within the lower bound $\boldsymbol{X}^L$ and upper bound $\boldsymbol{X}^U$ so to construct a feasible design space.

The evolutionary algorithm (EA) based on the biological evolution mechanism and Darwin's survival-of-the-fittest theory, was initiated at 60's in Europe [1] and Fogel et al. [2] in the U.S. Now EA has been extensively studied and applied to a wide range of applications and engineering designs. Over last few years there had been ever increasing interests in the area of artificial immune system (AIS) and their applications. Based on the theory of AIS to develop the adaptable optimization algorithm for engineering designs is a relatively new idea and progress, as described in the book of Corne et al. [3]. Inside Corne's book, one paper by Hajela and Yoo [4] presented an immune system (IS) modeling on the basis of genetic algorithm (GA) for design optimization to improve the GA's convergence and to handle design constraints. These applications to structural optimization can be sequentially found in Hajela's previous works [5–7]. Yoo and Hajela [7] presented the AIS concept as an alternative multi-criterion approach where multiple points on the Pareto front can be simultaneously generated in a single GA based simulation using the operations of selection, crossover and mutation. Coello Coello and Cortés [8] extended the work of Hajela and Lee [5] proposed an-

*Corresponding author. E-mail: cjs@mail.tku.edu.tw

other parallel version [9] coupled a GA to obtain the higher efficiency for constraints handling. Most developments using the AIS stated above promote the local search ability of optimization.

Various approaches of evolutionary multi-objective optimization were developed during last decade [10]. Yoo and Hajela's work [7] may be the pioneer of applying the immune theory in GA to obtain the Pareto front in two-objective structural optimization problems. The other important development in this area by Coello Coello and Cortés [11−13] in which the high affinity antibodies will proliferate based on the clonal selection principle. A good Pareto front of five test functions with simple constrained functions can be obtained through a mechanism containing the adaptive grid, secondary memory, duplication, mutation and a fraction of cross over. Luh et al. [14] proposed another immune algorithm for obtaining the Pareto front in unconstrained two-objective optimization and verified it with six test problems.

The enhanced expression strategy modified from Hajela and Yoo's GA [15] is applied to constraints handling and to performance improvement in this immune based multi-objective evolutionary algorithm (IMEA). The enhanced strategy had ever been successfully applied to the immunity based hybrid evolutionary algorithm for single objective engineering optimization [16]. However, the multiple objective design problems have not been well explored yet. This paper aims at presenting an efficient development of the hybrid evolutionary optimization algorithm inspired from the immune system that can directly treat constraints by enhanced expression strategy for multiple objective design optimization problems.

2. The Immune System and Inspirations

The architecture of the immune system is multilayered, with defenses on several levels. Once pathogens have entered the body, they are dealt with by the innate immune system and the adaptive immune system. Two aspects of the IS must face: 1. The identification or detection of pathogens means to distinguish the harmful nonself and everything else. 2. The efficient elimination of those pathogens while minimizing harm to the body. Different pathogens have to be eliminated in ways by choosing the right effectors for the particular kind of pathogens. The adaptive IS adapts or learns to recognize specific kinds of pathogens, and retains a memory of them for speeding up future responses. Pathogens have many different epitopes so many different lymphocytes may be specific to a single kind of pathogen. The strength of the bond between a receptor and an epitope is termed the affinity. The IS must have a sufficiently diverse repertoire of lymphocyte receptors to ensure that at least some lymphocytes bind to any given pathogen.

A class of lymphocytes called B-cells implements the both of these two principles. The B-cell produces many clones in the lymph node and this cloning is subject to a form of somatic hyper-mutation. If new B-cells succeed in binding to pathogenic epitopes, they will leave the lymph node and differentiate into plasma or memory B-cells. Plasma B-cells secrete a soluble form of their receptors, called antibodies. Figure 1 shows a conceptual clonal selection principle of B-cells [17]. This activation-proliferation-differentiation cycle is repeated and results in increasing selection of high-affinity B-cells, as called affinity maturation.

Now let us consider a general unconstrained optimization problem: find $\boldsymbol{X}^* (= x_1^*, x_2^*, \ldots, x_n^*)$ by minimizing $f(\boldsymbol{X})$. How can one apply the IS on it? The $\boldsymbol{X}^*$ is simulated by a single pathogen with n specific epitopes corresponding to the minimum $f(\boldsymbol{X}^*)$, is called the antigen. An antibody population expresses in $[\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_N]^T$ where each $\boldsymbol{X}_i$ is simulated as an antibody with n receptors. In the sense of numerical simulation, this can be initially random-generated and then proliferate to diverse distribution that imitates a recombination of DNA results in different B-cells genes, and hence different receptors. Those antibody populations then go through a matching process to evaluate the degree of fitting to $\boldsymbol{X}^*$ for a further variable selection. One can be inspired to further develop non-inferior solutions for multiple design objectives.

3. Multi-Objective Optimization

A true solution obtained from multi-objective approach is a trade-off that indicates the improvement in any one objective would adversely affect another objective. Since the solutions satisfy this statement has unlimited numbers so that to generate a front of compromise designs becomes a natural mission in solving a general multi-objective optimization problem. Such a front represents a set of Pareto optimal designs on which each

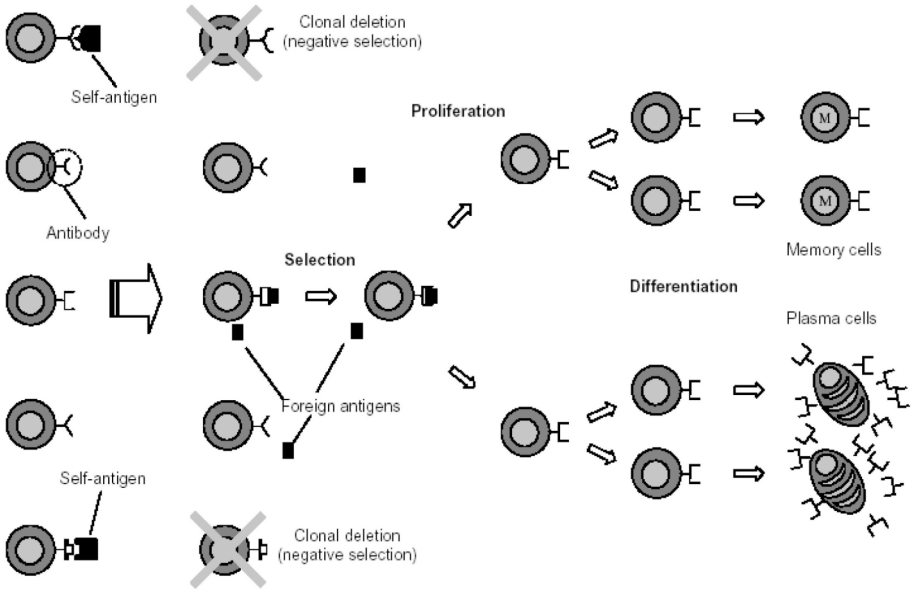

Figure 1. Clonal selection principle of B-cells (From [17]).

point represents a particular Pareto point. The Pareto front (Pareto solution curve on the boundary of feasible domain) of two-objective functions can be illustrated in Figure 2 where both $f_a(\boldsymbol{X})$ and $f_b(\boldsymbol{X})$ are minimized simultaneously. Point A indicates a point inside the feasible domain. Each point such as point B or C along Pareto solution curve indicates a particular Pareto optimal design. The weighted-sum approach, associates a real weight ϖ_i with the ith objective f_i, is a weighting function method [18] of dealing with multi-objective optimization problems. Accumulating the weighted objective values yields the combined objective function expressed as:

$$\text{Minimize} \quad s(\boldsymbol{X}) = \sum_{i=1}^{M} \varpi_i f_i(\boldsymbol{X}) \tag{3}$$

$$\sum_{i=1}^{M} \varpi_i = 1 \tag{4}$$

The resulting design associated to each set of weighting coefficient (ϖ_i) corresponds to a particular Pareto design. Therefore, many set of weighting schedule is able to construct the Pareto optimal front. When one observes Eq. (3) in which the objective function ($f_i(\boldsymbol{X})$, $i = 1, 2, \ldots, M$) can be much different from each other. Therefore, it is not always appropriate by simply multiplying the weighting parameter and then summarizes them. The experiences show that the formulation in Eq.

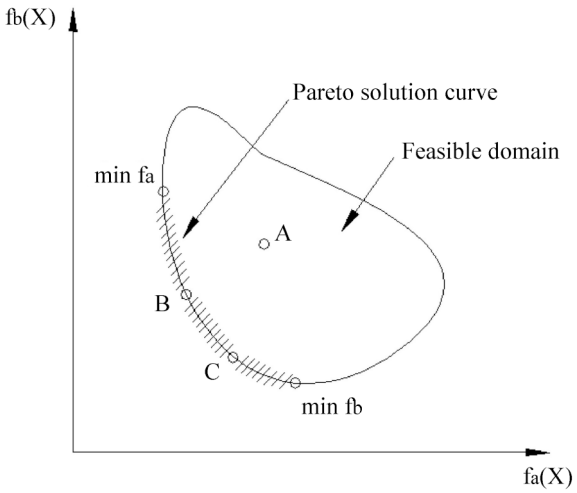

Figure 2. Pareto front solutions of two-objective functions

(3) may yields to the phenomenon of premature and of unstable convergence. In this paper, we apply an alternative formulation in stead of Eq. (3) such that:

$$s(\boldsymbol{X}) = \sum_{i=1}^{M} \varpi_i \left(\frac{f_i(\boldsymbol{X}) - f_i(\boldsymbol{X}^{id})}{f_i(\boldsymbol{X}^{id})} \right) \tag{5}$$

where $f_i(\boldsymbol{X}^{id})$ is the ith objective function associated to the ith ideal solution, usually it can be obtained by minimizing individual $f_i(\boldsymbol{X})$ subject to the original constraints. In practical sense, each $f_i(\boldsymbol{X}^{id})$ can be approxi-

mately equal to or less than the minimum value of f_i.

4. Immune Based Multi-Objective Evolutionary Algorithm (IMEA)

A complete hybrid immunity-based evolutionary multi-objective optimization algorithm consisting of the enhanced expression strategy is described as follows.

1. Initialization
   1.1 Assign n_ϖ sets of weighted-coefficient, and n_ϖ set of composite function $s_i(\boldsymbol{X})$, $i = 1, 2, \ldots, n_\varpi$, can be established. Define each fitness function $s_i(\boldsymbol{X})$, $i = 1, 2, \ldots, n_\varpi$, is the antigen function satisfies all constraints ($g_i(\boldsymbol{X}) \le 0$, $i = 1, 2, \ldots, m$) in Eq. (1). Select the real-coded representation, mutation rate r_m and number of antibody population size N.
   1.2 Uniformly and randomly generate initial N individuals as original candidate-antibodies in the population.
   1.3 Compute the fitness value for each objective, expressed as ($s_i(\boldsymbol{X}_i)$, $j = 1, 2, \ldots, N$), $i = 1, 2, \ldots, n_\varpi$). Compute all constrained functions ($g_i(\boldsymbol{X})$, $j = 1, 2, \ldots, m$) and their violations corresponding to each $s_i(\boldsymbol{X}_i)$. Select and memorize the best feasible individual $(\boldsymbol{X}_b)_i$, $i = 1, 2, \ldots, n_\varpi$, where the best one indicates that possesses the highest fitness corresponding to the ith weighting formulation. Set the starting generation t is zero.

2.Expression

All infeasible designs were ranked on the basis of the violation to constraints, with a higher rank given to the more infeasible design.

Define the a representation $(\delta_{IJ})_i$, $i = 1, 2, \ldots, n_\varpi$, expressed as following:

$$\delta_{IJ} = O_b(x_I) - O_b(x_J) \tag{6}$$

Then the feasible design I yields the smallest absolute value of δ_{IJ} was selected for the expression operation with the Jth infeasible one. However, the negative value of δ_{IJ} is preferred over a positive δ_{IJ} even if the absolute value of the latter was smaller.

Once n_ϖ set of infeasible designs in the population are identified based on Eq. (6), the enhanced expression operation (Eq. 7) is carry out on a bit-by-bit basis as previous single optimization algorithm.

$$x_{ij}^E = \begin{cases} (x_I)_i & \text{if } r_i < p_j \\ x_{ij}^E & \text{if } r_i \ge p_j \end{cases} \quad j = 1, 2, \ldots, m_{in} \tag{7}$$

3. Proliferation
   3.1 Recombination

   Randomly select two individuals using multi-point crossover strategy to reproduce two offspring on the bit-by-bit basis. Select the one with the better fitness than $(\boldsymbol{X}_b)_i$ and replace the previous $(\boldsymbol{X}_b)_i$, $i = 1, 2, \ldots, n_\varpi$, that has been defined in step 1.3.
   3.2 Mutation

   The number of $r_m \times N \times n$ individuals will perform mutation operation using a mutation rate of r_m. The one with the higher fitness is adopted to replace the previous $(\boldsymbol{X}_b)_i$, $i = 1, 2, \ldots, n_\varpi$, as compared in step 1.3 and 3.1.

4. Differentiation
   4.1 Select the $(\boldsymbol{X}_b)_i$ with the highest fitness as the ith antigen ($i = 1, 2, \ldots, n_\varpi$).
   4.2 In the antibody population, a random number n_s is selected to perform the antibody-antigen matching process from $(\boldsymbol{X}_b)_i$.

$$z_i = s_i(\boldsymbol{X}) - s((\boldsymbol{X}_b)_i), \quad (i = 1, 2, \ldots, n_\varpi) \tag{8}$$

The antibody with the highest affinity is retained and then drops it into another pool.
   4.3 Repeat the previous step until the number of antibodies in another pool of previous step is (N-n_ϖ).

5. Examination and termination

While the evolving value of the best antibody is steady consecutively in repeating steps 2 to 5, the searching process is terminated. The best antibody of each $s_i(\boldsymbol{X})$, ($i = 1, 2, \ldots, n_\varpi$) is selected to construct the Pareto optimum set. Otherwise, let $t = t + 1$, go to step 2 and the next generation evolution is continuously carry out.

5. Illustrative Engineering Problem

5.1 Symmetric Three-Bar Truss Design

A symmetric three-bar truss shown in Figure 3 is considered by minimizing two different objectives to find

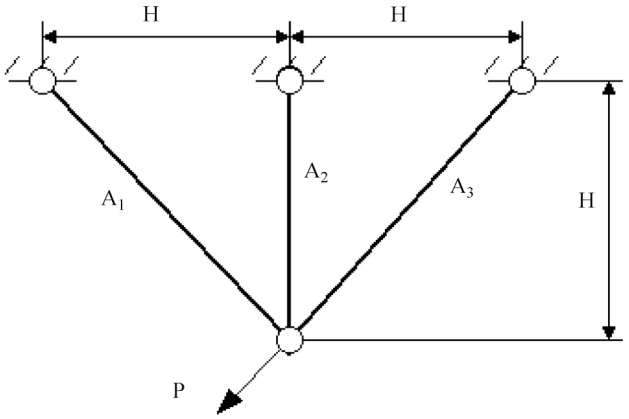

Figure 3. A symmetric three-bar truss with loading.

the optimum cross-sectional areas of members 1 (and 3) and 2. The complete explicit formulation and information can be found in [19,20]. Let P = 20, H = 1, $E = 1$ and $\rho = 1$. This optimization problem can be stated as: Find $\boldsymbol{X} = [x_1, x_2]^T = [A_1, A_2]^T$ that minimizes weight represented as $f_1(\boldsymbol{X}) = 2\sqrt{2}x_1 + x_2$ and vertical deflection of loaded joint represented as $f_2(\boldsymbol{X}) = PH / (E \times (x_1 + \sqrt{2}x_2))$. Design constraints are the stress induced in each member that must be restricted in the range of $x_1 \le 5$, $i = 1, 2$.

Eleven ($n_\varpi = 11$) different weighting combinations were considered here which results in a total 11 antigen functions, such that $s_i(\boldsymbol{X})$, $i = 1, 2, ..., 11$. This problem was solved by the proposed IMEA with enhanced expression strategy for constraints handling, in which the total population is 200, random number n_s of antibody is 50, and the mutation rate is 0.1. The final numerical design is listed in Table 1 corresponding to 11 set of weighting combinations. Each optimum $\boldsymbol{X}^*$ represents a Pareto design in Pareto solution curve. The result of two objectives can make a plot in Figure 4 that represents the Pareto front of optimal designs. Another plot of ϖ_1 (or $\varpi_2 = 1 - \varpi_1$) against $s(\boldsymbol{X}^*)$ shown in Figure 5 indicates the Pareto front from another view. This two-objective optimum result was further compared with five designs in [20] solved by the alternative global criterion method

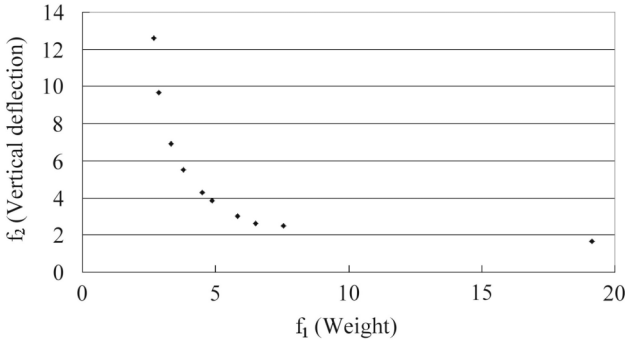

Figure 4. Pareto front curve of symmetric truss design using IMEA.

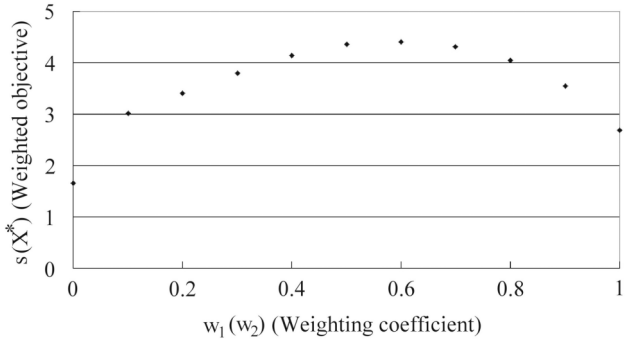

Figure 5. Another view of the Pareto front of symmetric truss design using IMEA.

Table 1. Pareto design of symmetric truss problem using IMEA

Weighting Coeff. (ϖ_1, ϖ_2)	Design Variables $\boldsymbol{X}^* = (x_1, x_2)$	Weight $f_1(\boldsymbol{X}^*)$	Vertical Deflection $f_2(\boldsymbol{X}^*)$	Weighted Objective $s(\boldsymbol{X}^*)$
0.0, 0.1	4.9990, 5.0000	19.1393	1.6569	1.6569
0.1, 0.9	0.9060, 5.0000	7.5625	2.5072	3.0127
0.2, 0.8	0.5354, 5.0000	6.5143	2.6293	3.4063
0.3, 0.7	0.5352, 5.0000	6.5136	2.6295	3.7947
0.4, 0.6	0.5410, 4.2910	5.8218	3.0260	4.1441
0.5, 0.5	0.5572, 3.2999	4.8759	3.8285	4.3522
0.6, 0.4	0.5600, 2.9100	4.4939	4.2777	4.4074
0.7, 0.3	0.5800, 2.1600	3.8005	5.5025	4.3111
0.8, 0.2	0.6060, 1.6119	3.3259	6.9310	4.0470
0.9, 0.1	0.6600, 0.9990	2.8658	9.6488	3.5441
1.0, 0.0	0.7400, 0.6000	2.6931	12.5901	2.6931

(AGCM), displayed in Table 2 [20]. In addition to the consideration of computation cost, efficiency and convergence, when one compares the value of $s(\boldsymbol{X}^*)$ in Table 1 and Table 2, five set of optimal design in Table 1 appears better performance than that in Table 2.

5.2 Simply-Supported I-Beam Structural Design

A simply supported I-beam structure was designed simultaneously to minimize both structural weight (or cross-sectional area) and static deflection at the mid-span, as shown in Figure 6. The structure sustains a concentrated load P (134,880 lb) in vertical direction and a load Q (11,240 lb) in transverse direction. This beam has the Young's modulus 29.0×10^6 psi and the allowable bending stress 23,205 psi. The dimensions of the structure expressed as x_i (i = 1, 4) represents four design variables. The two-objective design optimization problem can be stated as: to minimize the cross-sectional area expressed as $f_1(\boldsymbol{X}) = 2x_2x_4 + x_3(x_1 - 2x_4)$ and minimizes the deflection of mid-span expressed as $f_2(\boldsymbol{X}) = PL^3/48EI(\boldsymbol{X})$. All explicit functions and information can be found in [7,21].

Eleven (n_ϖ = 11) different weighting combinations were considered that represent 11 antigen functions, such that $s_i(\boldsymbol{X})$, $i = 1, 2, \ldots, 11$. This problem was solved by the proposed approach of IMEA in which function $s(\boldsymbol{X})$ is expressed in the form of Eq. (5); the total population is 200, random number n_s of antibody is 50, and the mutation rate is 0.1. The final numerical design listed in Table 3 is almost the same as the results of GA coupled

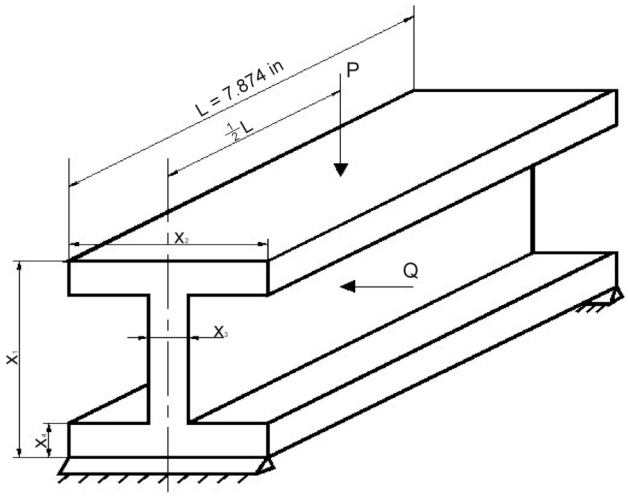

Figure 6. A simply supported I-beam structure.

Table 2. Pareto design of symmetric truss problem using AGCM [20]

Weighting Coeff. (ϖ_1, ϖ_2)	Weight $f_1(\boldsymbol{X}^*)$	Vertical Deflection $f_2(\boldsymbol{X}^*)$	Weighted Objective $s(\boldsymbol{X}^*)$
0.3, 0.7	6.5233	2.7061	3.8513
0.4, 0.6	5.9370	3.0423	4.2002
0.5, 0.5	5.4163	3.4107	4.4135
0.6, 0.4	4.9587	3.8477	4.5143
0.7, 0.3	4.4760	4.3656	4.4429

Table 3. Pareto design of I-beam problem using proposed IMEA

Weighting Coeff. (ϖ_1, ϖ_2)	Design variables $\boldsymbol{X}^* = (x_1, \ldots, x_4)$	Weight $f_1(\boldsymbol{X}^*)$	Vertical Deflection $f_2(\boldsymbol{X}^*)$	Weighted Objective $s(\boldsymbol{X}^*)$
0.0, 0.1	31.4960,19.6839,1.9546,1.9684	131.3577	0.00233	0.0023
0.1, 0.9	31.4960,19.6849,0.3546,1.9685	87.2705	0.00270	8.7295
0.2, 0.8	31.4960,19.6839,0.3546,1.9680	87.2479	0.00270	17.4517
0.3, 0.7	31.4960,19.6839,0.3545,1.4850	68.5758	0.00341	20.5751
0.4, 0.6	31.4960,19.6839,0.3545,1.1732	56.5220	0.00417	22.6113
0.5, 0.5	31.4960,19.6839,0.3545,0.9420	47.5841	0.00502	23.7945
0.6, 0.4	31.4960,19.6800,0.3545,0.7510	40.1918	0.00608	24.1175
0.7, 0.3	31.4960,19.6800,0.3545,0.4234	33.7371	0.00749	23.6182
0.8, 0.2	31.4960,19.6800,0.3545,0.4234	27.5297	0.00971	22.0257
0.9, 0.1	31.4950,13.9839,0.3545,0.3571	20.8984	0.01441	18.8100
1.0, 0.0	29.3402,14.2797,0.3545,0.3546	20.2767	0.01677	20.2767

with immune application by Yoo and Hajela [7]. The information of two objectives can produces a plot shown in Figure 7 that represents the Pareto front of optimal design. The plot of ϖ_1 against $s_i(\boldsymbol{X}^*)$ in Figure 8 shows another view of the Pareto front.

The two numerical optimization problems show that the proposed IMEA is effective for constructing the Pareto front in a single simulation. The points between two Pareto points can be consequently obtained by the numerical interpolation technique. Although the solution efficiency, convergence and computation cost are not considered particular interests at this stage. One is able to observe that the proposed IMEA is primarily based on the immune system theory that is different from the GA based algorithm [7] and different from the gradient based algorithm [20].

6. Conclusions

A hybrid evolutionary algorithm named IMEA based on simulating the immune system theory is presented, as an alternative algorithm for dealing with multi-objective engineering optimization problems. The proposed approach has features of using the real-number coded representation, the principle of affinity maturation in immune system, GA's recombination and the enhanced expression strategy for constraints handling. The proposed multi-objective optimization is successful to obtain the Pareto front during a single run computation. A normalization expression used in mathematical form can avoid the premature during the evolutionary computation that results in a stable convergent characteristic. Two double-objective design optimization problems solved by proposed IMEA show that the Pareto design can be noticeably obtained, as compared to published papers [7,20]. Moreover, some results by IMEA are precisely obtained than that by the gradient based optimization approach [20].

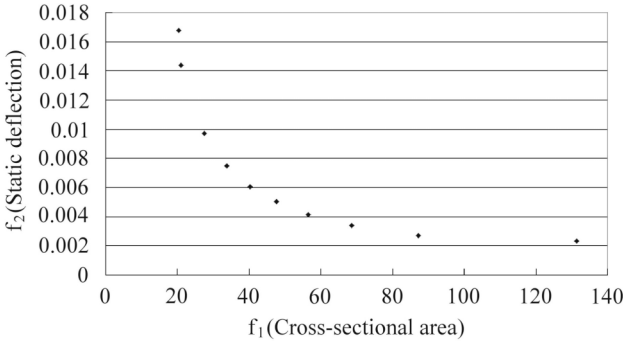

Figure 7. Pareto front curve of I-beam structural design using IMEA.

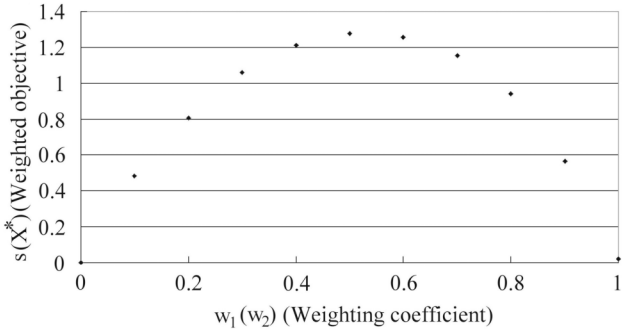

Figure 8. Another view of Pareto front for I-beam design using IMEA.

Acknowledgement

The support received from the National Science Council, Taiwan under Grant NSC 94-2212-E-032-002 and 95-2221-E-032-007, is gratefully acknowledged.

References

[1] Box, G. E. P., "Evolutionary Operation: A Method for Increasing Industrial Productivity," *Applied Statistics,* Vol. 6, pp. 81–101 (1957).

[2] Fogel, L. J., Owens, A. J. and Walsh, M. J., *Artificial Intelligence Through Simulated Evolution*. John Wiley, New York (1966).

[3] Corne, D., Dorigo, M. and Glover, F. (Eds), *New Ideas in Optimization,* McGraw-Hill International Limited, UK (1999).

[4] Hajela, P. and Yoo, J. S., "Immune Network Modelling in Design Optimization," *New Ideas in Optimization*, Editors: Corne, D., Dorigo, M. and Glover, F., McGraw-Hill International, UK (1999).

[5] Hajela, P. and Lee, J., "Constrained Genetic Search via Schema Adaptation: An Immune Network Solution," *Structural Optimization,* Vol. 12, pp. 11–15 (1996).

[6] Lee, J. and Hajela, P., "GA's in Decomposition Based Design-Subsystem Interactions Through Immune Network Simulation," *Structural Optimization,* Vol. 14, pp. 248–255 (1997).

[7] Yoo, J. and Hajela, P., "Immune Network Simulations in Multi-Criterion Design," *Structural Optimization,*

Vol. 18, pp. 85–94 (1999).

[8] Coello Coello, C. A. and Cortés, N. C., "Use of Emulations of the Immune System to Handle Constraints," *Evolutionary Algorithms, Intelligent Engineering Systems Through Artificial Neural Networks* (*ANNIE' 2001*), ASME Press, Editors: Dagli, C. H. et al., Vol. 11, pp. 141–146 (2001).

[9] Coello Coello, C. A. and Cortés, N. C., "A Parallel Implementation of an Artificial Immune System to Handle Constraints in Genetic Algorithms: Preliminary Results," *Proceedings of the Congress on Evolutionary Computation 2002*, IEEE Service Center, Vol. 1, pp. 819–824, Piscataway, New Jersey (2002).

[10] Coello Coello, C. A., http://www.lania.mx/~ccoello /EMOO/EMOObib.html. (2005).

[11] Coello Coello, C. A. and Cortés, N. C., "An Approach to Solve Multi-objective Optimization Problems Based on an Artificial Immune System," *First International Conference on Artificial Immune Systems* (*ICARIS'2002*), Jonathan Timmis, J. and Bentley, P. J. (editors), pp. 212–221, University of Kent at Canterbury, Inglaterra (2002).

[12] Coello Coello, C. A. and Cortés, N. C., "Solving Multi-Objective Optimization Problems Using an Artificial Immune System," *Technical Report EVOCINV*-05-2002, Evolutionary Computation Group at CINVESTAV, Seccion de Computacion, Departamento de Ingenieria Electrica, CINVESTAV-IPN, Mexico (2002).

[13] Cortés, N. C. and Coello Coello, C. A., "Using Artificial Immune Systems to Solve Optimization Problems," Barry, A. (editor), *Genetic and Evolutionary Computation Conference*. Workshop Program, pp. 312–315, AAAI, Chicago, Illinois, USA (2003).

[14] Luh, G.-C., Chueh, C.-H. and Liu, W.-W., "MOIA-Multi-Objective Immune Algorithm," *Engineering Optimization*, Vol. 35, pp. 143–164 (2003).

[15] Hajela, P. and Yoo, J., "Constraint Handling in Genetic Search Using Expression Strategies," *AIAA Journal,* Vol. 34, pp. 2414–2420 (1996).

[16] Shih, C. J. and Kuan, T. L., "An Immunity Based Hybrid Evolutionary Algorithm for Engineering Optimization," *Tamkang Journal of Science and Engineering - An International Journal,* Vol. 9, pp. 25–36 (2006).

[17] de Castro, L. N. and Timmis, J., "An Artificial Immune Network for Multi-Modal Function Optimization," *Proceedings of IEEE Congress of Evolutionary Computation* (*CEC'02*), Vol. 1, pp. 699–674 (2002).

[18] Rao, S. S., *Engineering Optimization - Theory and Practice.* 3rd ed., Wiley Eastern Limited, New York (1996).

[19] Rao, S. S., "Multi-Objective Optimization of Fuzzy Structural Systems," *International Journal of Numerical Methods in Engineering,* Vol. 24, pp. 1157–1171 (1987).

[20] Shih, C. J. and Chang, C. J., "Pareto Optimization of Alternative Global Criterion Method for Fuzzy Structural Design," *Computers & Structures,* Vol. 54, pp. 455–460 (1995).

[21] Hajela, P. and Shih, C. J., "Multi-Objective Optimum Design in Mixed Integer and Discrete Design Variable Problems," *AIAA Journal,* Vol. 28, pp. 670–675 (1990).

Manuscript Received: Jul. 30, 2007
Accepted: Jul. 17, 2008